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Coiled Tubing Hydraulics Modeling

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Contents

Summary

This document presents a general formulation of the governing equation used to determine the system pressure losses for liquids, gases, and multiphase fluids. Both CT and annulus flow are considered.

Introduction

In most coiled tubing (CT) applications such as cleanouts, well unloading, acidizing, stimulation, drilling, etc., fluid (liquid, gas, or multiphase mixture of liquid and gas) is pumped through the CT to a desired depth in the wellbore, and returned up the annulus. The fluid returning in the annulus that is formed between the CT and production tubing/casing can be a multiphase mixture of the pumped CT fluid, reservoir fluid, original wellbore fluid, and sand or drilled solid particles, depending on the application. Water, air, nitrogen, diesel, brines, acids, gels, and foams are among the many commonly pumped fluids through CT in various applications. Thus, depending on the fluid type and properties, system pressures (pump pressure, gooseneck pressure, wellhead pressure, flowing bottom hole pressure) change and affect the pumping requirements. In addition, system pressures are also affected by many other parameters such as pump rate, CT size (length, diameter), reel core diameter, geometry of the wellbore, and surface roughness.

From a fluid mechanics viewpoint, flows that occur during any CT application can be broadly classified as:

- compressible (fluid density is a strong function of pressure such as in gases) or incompressible (fluid density is a very weak function of pressure such as for liquids)
- steady (flow is independent of time) or unsteady (flow is time-dependent)
- laminar (flow is characterized by layers or streamlines) or turbulent (flow is characterized by random mixing and is no longer streamlined)
- single-phase (only one fluid phase exists as either liquid or gas) or multiphase (at least two phases exist as in liquid and gas; or liquid, gas, and solid)
- Newtonian (shear stress is linearly related to shear rate) or non-Newtonian (shear stress is not linearly related to shear rate)
- upwards or downwards in a vertical, inclined, or horizontal wellbore
- in the CT or in the annulus formed between the CT and production tubing/casing.

In order to accurately predict the system pressures in any CT operation, all these fluid mechanics aspects and parameters must be taken into careful consideration during the mathematical development of a wellbore hydraulics model for CT applications.

System pressures can be estimated by appropriately accounting for the total pressure losses (ΔP) in the CT and annulus. In general, the total pressure loss is comprised of three components:

- hydrostatic pressure loss (ΔP_h),
- friction pressure loss (ΔP_f), and
- acceleration pressure loss (ΔP_a).

The magnitude of each of these components is largely affected by fluid properties such as density and viscosity, and the total pressure loss can be accurately predicted only if the fluid properties are evaluated correctly. Moreover, the frictional component of the coiled tubing pressure loss can be further subdivided into two parts: a part that accounts for the friction pressure loss on the reel (ΔP_{CT}) before entering the well, and a part that accounts for the straight tubing losses (ΔP_{ST}) in the well. Experiments [McCann and Islas (1996); Azouz *et al.* (1998)] have shown that, in general, coiled tubing pressure losses are greater than the corresponding straight tubing pressure losses, and therefore must be accounted for appropriately while determining the system pressure losses.

This document presents a general formulation of the governing equations to determine the system pressure losses for various fluids (liquids, gases, foams, and multiphase fluids). The analysis considers both CT and annulus flow. The evaluation of fluid properties, such as density and viscosity, for each of these fluid types is discussed in detail with an emphasis on the influence on the system pressure losses. In addition, criteria for the onset of turbulence and evaluation of friction factors are discussed for all fluid types considered. Furthermore, correlations that determine the friction factor in reeled coiled tubing (f_{CT}) in relation to the friction factor (f_{ST}) for straight tubing are also presented for some fluid types.

CT and Annular Flow

CT Flow

The governing equation for system pressure losses stems from the mechanical energy balance, and is written for a coiled tubing segment as shown in Figure 1.

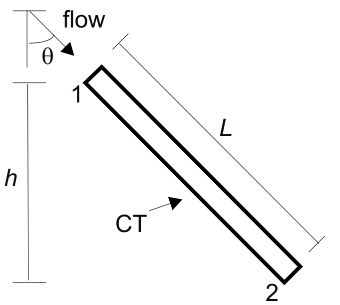

FIGURE 1 CT segment of length L and inclined at an angle θ to the vertical

Thus, the total pressure loss ($\Delta P^s = P_2 - P_1$, where P_1 and P_2 are the pressures at points 1 and 2 respectively) for downward flow through a CT segment of length L inclined at an angle θ to the vertical is given by

$$\Delta P^s = \Delta P_h^s - \Delta P_f^s - \Delta P_a^s \tag{Eq 1}$$

where the hydrostatic (ΔP_h^s), frictional (ΔP_f^s), and acceleration (ΔP_a^s) components can be written as

$$\Delta P_h^s = \frac{\rho g h}{g_c} \tag{Eq 2}$$

$$\Delta P_f^s = f \, \frac{\rho v^2 L}{2 g_c D_e} \tag{Eq 3}$$

and

$$\Delta P_a^s = \frac{\rho}{2 g_c}(v_2^2 - v_1^2) \tag{Eq 4}$$

In Eq 1 through Eq 4, superscript s refers to segment; h is the vertical distance between points 1 and 2 and is given by $h = L\cos\theta$; g is the acceleration due to gravity; g_c is the Newton's law conversion constant for the English system of units; f is the Moody's friction factor; D_e is the equivalent diameter of the conduit (inside diameter of CT); ρ is the mean density of the fluid between points 1 and 2; v is the mean velocity of the fluid between points 1 and 2; and, v_1 and v_2 are the fluid velocities at points 1 and 2 respectively. It should be noted for horizontal flow ($\theta = 90$ degrees) in the CT, ΔP_h^s is zero, and only the frictional and acceleration contributions to the pressure loss exist. In addition, Eq 1 through Eq 4 and all subsequent equations in this document are written in consistent units unless otherwise noted.

The friction factor in Eq 3 is a function of the Reynolds number (Re), defined physically as the ratio of the inertia force to the viscous force. Mathematically, the definition of Re varies depending on the fluid type (e.g., Newtonian liquids, non-Newtonian liquids, gases, multiphase fluids), and therefore will be presented separately for each of the fluid types. In addition, the magnitude of Re distinguishes a flow from being in the laminar, transitional, or turbulent flow regimes. Thus, the friction factor is found as a function of Re for all these flow regimes in various fluid types. Similarly, computation of fluid properties (density, viscosity) differ depending on fluid type and will be discussed in separate sections.

Finally, since Eq 1 through Eq 4 are defined for a single CT segment, the total pressure loss (ΔP) in the CT can be found by a summation over the entire CT length as

$$\Delta P = \Delta P_h - \Delta P_f - \Delta P_a \tag{Eq 5}$$

where

$$\Delta P_h = \sum_1^N \Delta P_h^s \tag{Eq 6}$$

$$\Delta P_f = \sum_1^N \Delta P_f^s \tag{Eq 7}$$

and

$$\Delta P_a = \sum_1^N \Delta P_a^s \tag{Eq 8}$$

Here, N is the total number of segments in the CT.

Annular Flow

Although Figure 1 only depicts flow in a CT, the same analysis [Eq 1 through Eq 8] holds true for upward flow ($\pi/2 < \theta \leq \pi$)) in an annulus formed between the CT and production tubing/casing, provided the friction factor, f is evaluated based on the equivalent diameter, D_e. Several definitions are available for D_e in the literature, and, in general, can be written as

$$D_e = K_a(d_C - D_T)$$ **Eq 9**

where K_a is an annulus constant, d_c and D_T are the inner diameter of the production tubing/casing and outer diameter of the CT respectively. The most common values of K_a are 1.0 and 0.816. When $K_a = 1$, Eq 9 reduces to the hydraulic diameter (defined as four times the cross-sectional flow area divided by the wetted perimeter), and for a K_a value equal to 0.816, Eq 9 becomes the slot flow representation of an annulus flow. It should be noted that this slot flow approximation yields accurate results only for D_r / d_c ratios greater than 0.3 (small annular areas) and should be used with caution.

In the next few sections, computation of pressure losses in various fluid types is considered separately. The equations presented in the following sections are used in conjunction with the general formulation [Eq 1 through Eq 9] in order to determine the system pressures for a particular fluid type.

Pressure Losses in Liquids

Liquids are most often pumped through CT. Examples of commonly pumped liquids are fresh water, seawater, brines, acids, kerosene, crude oil, diesel, polymer gels, and drilling mud. These fluids can be broadly classified as Newtonian and non-Newtonian liquids. Non-Newtonian liquids can be further subdivided into many categories depending on the rheological model that best describes their fluid behavior. The most widely accepted non-Newtonian models in the petroleum industry are the Power-Law and Bingham Plastic models, and as such, these models will be discussed in some detail along with the Newtonian case.

Newtonian Model

Fluids that exhibit a linear relationship between the shear stress (τ) and shear rate (γ) are called Newtonian fluids. Mathematically, the relationship can be described as

$$\tau = \mu\gamma$$ **Eq 10**

where μ is the viscosity of the fluid. In general, liquid viscosity is a strong function of temperature and decreases with increasing temperature. Water, brines, acids, and light oils are good examples of Newtonian liquids.

The flow is usually characterized by the Reynolds number, which for Newtonian fluids can be written as

$$\mathrm{Re} = \frac{\rho v D_e}{\mu} \tag{Eq 11}$$

For internal flow of Newtonian fluids through straight conduits (including pipes and annuli), the flow can be classified as either laminar, transitional, or turbulent depending on the magnitude of the Reynolds number. The flow is laminar if Re is less than or equal to a critical value of 2100. A transitional flow is observed between Reynolds numbers of 2100 and 4000. If Re is greater than 4000, then the flow is turbulent. The friction factor for straight tubing (f_{ST}) in laminar flow is given by

$$f_{ST} = \frac{64}{\mathrm{Re}} \tag{Eq 12}$$

In turbulent flow, f_{ST} for smooth (Blasius equation) and rough pipes (Colebrook equation) can be expressed as [see Bourgoyne, Jr., *et al.* (1991)]

$$f_{ST} = \frac{0.3164}{\mathrm{Re}^{0.25}} \quad \text{(smooth pipes)} \tag{Eq 13}$$

$$f_{ST} = \left[\frac{1}{2}\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f_{ST}}} + \frac{\varepsilon / d_T}{3.715}\right)\right]^2 \quad \text{(rough pipes)} \tag{Eq 14}$$

In Eq 14, ε is the absolute roughness of the CT and is approximately equal to 0.00186 in. for commercial steel pipes.

However, in the case of flow through coiled tubing on the reel, the presence of a secondary flow (commonly referred to as Dean's vortices) perpendicular to the main flow makes the characterization somewhat complicated. Instead of the Reynolds number, another dimensionless number called the Dean number (Dn) is used to characterize the flow, and is defined as

$$Dn = \mathrm{Re}\left(\frac{d_T}{D_{reel}}\right)^{0.5} \tag{Eq 15}$$

where d_r is the inner diameter of the CT and d_{reel} is the reel core diameter. Experiments [Srinivasan *et al.* (1970)] have suggested that the laminar and turbulent flow regimes for flow through CT can be distinguished by the following correlation for the critical Reynolds number (Re_{cr})

$$\mathrm{Re}_{cr} = 2100\left(1 + 12\sqrt{\frac{d_t}{D_{reel}}}\right) \tag{Eq 16}$$

For laminar flow through CT, the friction factor can be expressed in terms of f_{ST} [Berger *et al.* (1983)] as

$$f_{CT} = f_{ST}\,(0.556 + 0.0969\sqrt{Dn}\,) \tag{Eq 17}$$

Similarly, in turbulent flow, Sas-Jaworsky and Reed (1997) have recently provided a correlation that is an extension of Ito's (1959) work as,

$$f_{CT} = f_{ST} + 0.03\sqrt{\frac{d_t}{D_{reel}}} \tag{Eq 18}$$

Clearly, from Eq 17 and Eq 18, the coiled tubing friction factors are greater than the straight tubing friction factors by an amount specified by the reel curvature. Eq 12 through Eq 14, Eq 17, and Eq 18 are utilized in Eq 3 to compute the friction pressure losses.

Liquid density and viscosity are also an integral part of the pressure loss computations and must be accounted for appropriately. Liquids are generally treated as incompressible fluids and their density change with pressure can be considered negligible. However, temperature effects on both liquid density and viscosity are significant (especially viscosity) and cannot be neglected. The above discussion on liquid density and viscosity holds true for all liquids (Power-Law, Bingham-Plastic) and hence is not addressed in subsequent subsections.

Power-Law Model

Fluids that exhibit a non-linear relationship between shear stress and shear rate are said to be non-Newtonian. More specifically, if the non-Newtonian relationship can be described by a two-parameter model such as

$$\tau = \mu_a \gamma$$

EQ 19

where the apparent viscosity (μ_a) is given by

$$\mu_a = K\gamma^{n-1}$$

EQ 20

then the fluid is said to exhibit a Power-Law type behavior. Here, n and K are called the *flow behavior index* and the *consistency index* respectively. If $n = 1$, the fluid exhibits Newtonian characteristics and the viscosity is independent of shear rate. However, for $n < 1$ and $n > 1$, the fluid viscosity is dependent on shear rate, and are characterized by shear thinning and shear thickening behavior respectively. Most polymer gels used in the petroleum industry and drilling mud can be described by a shear thinning behavior.

The Reynolds number for Power-Law fluids is different from its Newtonian counterpart [Eq 11] and is quite frequently referred to as a generalized Reynolds number (Re_G). Mathematically, it can be expressed as

$$\mathrm{Re}_G = \frac{\rho v^{2-n} D_e^n}{2^{n-3} K \left(3 + \frac{1}{n}\right)^n}$$

EQ 21

It should be noted that when $n = 1$, Eq 21 reduces to the Newtonian Reynolds number given by Eq 11. The critical Reynolds number (Re_{Gcr}) for straight tubing flow of Power-Law fluids is [Dodge and Metzner (1959); Schuh (1964)]

$$\mathrm{Re}_{Gcr} = 3470 - 1370n$$

EQ 22

Thus, the flow is laminar if $\mathrm{Re}_G \le \mathrm{Re}_{Gcr}$ and transitional if $\mathrm{Re}_{Gcr} < \mathrm{Re}_G < 4270 - 1370n$. The flow is turbulent if $\mathrm{Re}_G > 4270 - 1370n$. For laminar flow of Power-Law fluids through straight tubes,

$$f_{ST} = \frac{64}{\mathrm{Re}_G}$$

EQ 23

In the case of turbulent flow [Schuh (1964)],

$$f_{ST} = \frac{a}{\mathrm{Re}_G^b} \qquad \textbf{Eq 24}$$

where

$$a = \frac{\log_{10} n + 3.93}{50} \qquad \textbf{Eq 25}$$

$$b = \frac{1.75 - \log_{10} n}{7} \qquad \textbf{Eq 26}$$

It should be noted that for the Newtonian case of $n = 1$, Eq 24 reduces to the Blasius equation for smooth pipes and hence, the analysis does not take pipe roughness into account.

For CT flows of non-Newtonian fluids, the critical Reynolds number at which the transition from laminar to turbulent flow takes place is not well addressed in the literature. However, a recent publication [McCann and Islas (1996)] outlines the correlation for turbulent friction factor for Power-Law fluids as

$$f_{CT} = \frac{1.06a}{\mathrm{Re}_G^{0.8b}} \left(\frac{d_T}{D_{reel}} \right)^{0.1} \qquad \textbf{Eq 27}$$

Eq 27 has been shown [McCann and Islas (1996)] to been in excellent agreement with experimental data obtained with power-law fluids.

Bingham Plastic Model

The relationship between shear stress and shear rate for a Bingham-Plastic fluid can be written as

$$\tau = \tau_y + \mu_p \gamma$$

EQ 28

where τ_y and μ_p are commonly referred to as the yield stress and plastic viscosity respectively. Like the Power-Law model, a Bingham Plastic fluid is non-Newtonian and the fluid behavior is described by two parameters. It should be noted that when yield stress is zero, Eq 28 reduces to the Newtonian form described by Eq 10. Many drilling fluids and foams are described by the Bingham-Plastic model.

The criteria for turbulence of a Bingham-Plastic fluid is dependent on another dimensionless number called the Hedstrom number. In consistent units, the Hedstrom number (*He*) is defined as [see Bourgoyne, Jr., *et al.* (1991)]

$$He = \frac{\rho \tau_y D_e^2}{\mu_p^2}$$

EQ 29

Hanks and Pratt (1967) demonstrated that *He* could be correlated with the critical Reynolds number, which determines the onset of turbulence. This correlation is obtained from the simultaneous solution of the following two equations

$$\frac{\left(\frac{\tau_y}{\tau_w}\right)}{\left(1 - \frac{\tau_y}{\tau_w}\right)^3} = \frac{He}{16800}$$

EQ 30

$$\mathrm{Re}_{cr} = \frac{1 - \frac{4}{3}\left(\frac{\tau_y}{\tau_w}\right) + \frac{1}{3}\left(\frac{\tau_y}{\tau_w}\right)^4}{8\left(\frac{\tau_y}{\tau_w}\right)} He$$

EQ 31

Here, τ_w is the wall shear stress. Eq 30 and Eq 31 are solved iteratively to determine the critical Reynolds number. The flow is laminar if $\mathrm{Re} < \mathrm{Re}_{cr}$ and turbulent if $\mathrm{Re} > \mathrm{Re}_{cr}$. The friction factors in straight and coiled tubing are evaluated just like in the Newtonian case [Eq 11 through Eq 18].

Pressure Losses in Gases

Air, nitrogen, and natural gas are frequently pumped through CT, with nitrogen used most often because of its inert properties. As mentioned earlier, gases are compressible and behave according to the real gas law, expressed as

$$\frac{P}{\rho} = ZRT \qquad \textbf{Eq 32}$$

where Z is the compressibility factor, R is the gas constant, and T is the temperature of the fluid. Using Eq 32 in the differential form of the mechanical energy balance, an expression can be derived for the friction pressure loss of gases in conduits. This expression is given by [McClain (1952)]

$$\Delta P_{fg}^{s} = P_1^2 - P_2^2 = f\left(\frac{L}{D_e}\right)\left(\frac{\dot{m}^2 ZRT}{g_c A^2}\right) \qquad \textbf{Eq 33}$$

where $\dot{m}$ is the mass flow rate of gas and A is the cross-sectional area of the conduit (pipe or annulus). Eq 33 is once again written in relation to the segment shown in Figure 1 and replaces Eq 3 in the set of governing equations when gas calculations are performed. Unlike liquids where the density is independent of pressure for all segment calculations, gas density is a strong function of pressure and is calculated iteratively at points 1 and 2 for each segment (see Figure 1). The mean density is simply the average of the densities at points 1 and 2 respectively.

Most gases are Newtonian in their fluid behavior and thus can be described by Eq 10. However, gas viscosity is a function of both pressure and temperature and therefore, must be accounted for appropriately. The viscosity of pure gases or gas mixtures may be estimated from the dimensionless reduced viscosity ($\mu^N = \mu/\mu_o$, where μ is the gas viscosity at a given pressure and temperature, and μ_o is the gas viscosity at atmospheric pressure and same temperature) plotted as a function of dimensionless reduced temperature and pressure [Carr *et al.* (1954)]. The reduced pressure and temperature can be evaluated from the critical properties of the gas. Another correlation that is quite popular in the petroleum industry for estimating the viscosity of natural gases is based on the work of Lee *et al.* (1966). Both these methods are used in Cerberus to estimate the gas viscosity.

In order to determine the gas density, viscosity, and pressures for each segment, the compressibility factor must be determined. The compressibility factor for air and nitrogen is calculated from correlations developed for the nitrogen Z-factor as a function of reduced pressure and temperature. The data from which these correlations are developed is provided in Sage and Lacey (1950). The *Z*-factor computation for natural gas is based on the Hall and Yarborough (1971) equation and is not discussed here.

Since most gases are Newtonian, the criteria for turbulence is similar to that of Newtonian fluids. It is defined in terms of the Reynolds number given by Eq 11 and replacing the liquid viscosity with a gas viscosity. The friction factors are evaluated from the straight tubing friction factors in Eq 12 through Eq 14. Currently, no friction factor correlation is available for the flow of gases in reeled CT, and the CT flow is modeled by assuming that the length of tubing on the surface is horizontal and straight.

Pressure Losses in Foams

Foams are essentially multiphase fluids and comprise of a mixture of liquid, gas, and a surfactant. Although foams are multiphase fluids, they are treated separately here because their rheological behavior has been observed to be similar to that of Bingham-Plastic fluids. These multiphase mixtures can be water-based or oil-based foams depending on the composition of the liquid medium. The gas phase is usually nitrogen, however, air and carbondioxide have also been used. The gas phase exists as microscopic bubbles and, in practice, may occupy between 10 to 95 percent of the total foam volume. The ratio of volume fraction of gas (V_g) to the total volume of foam (V) characterizes the foam in terms of its quality (q) defined mathematically as

$$q = \frac{V_g}{V}$$

EQ 34

Since gas is compressible, the quality of foam depends on both temperature and pressure. Thus, the foam quality is a varying parameter and must be calculated for each segment (see Figure 1) during the CT hydraulics simulation.

As mentioned earlier, foams have been treated as Bingham-Plastic fluids in the literature and, for practical purposes, their rheological behavior can be expressed in terms of an effective viscosity (μ_e) [Blauer *et al.* (1974)] shown below

$$\mu_e = \mu_{foam} + \frac{g_c \tau_y D_e}{6v}$$

EQ 35

where μ_{foam} is the plastic foam viscosity and is a function of q. For foam qualities less than 0.52, the gas exists as uniformly dispersed, non-interacting spherical bubbles in the liquid medium. For such cases, the plastic foam viscosity can be adequately described by Einstein's (1906) theory for a dilute suspension of rigid particles as

$$\mu_{foam} = \mu_l (1 + 2.5q) \tag{Eq 36}$$

where μ_l is the viscosity of the liquid phase. For qualities greater than 0.52 and less than 0.74, it has been found that the spherical gas bubbles interact with one another during flow. Hatschek (1910) proposed an expression for viscosity of interacting particles in this range of qualities as

$$\mu_{foam} = \mu_l (1 + 4.5q) \tag{Eq 37}$$

When the foam quality is above 0.74, the gas bubbles deform from spheres to parallelepipeds. The plastic foam viscosity for parallelepiped gas bubbles is given as [Hatschek (1910)]

$$\mu_{foam} = \mu_l \frac{1}{1 - q^{1/3}} \tag{Eq 38}$$

Eq 36 through Eq 38 are used in Eq 35 in order to evaluate the effective foam viscosity. In addition, the yield stress of the foam (τ_y) in Eq 35 is zero for qualities less than 0.52. However, for $q > 0.52$, τ_y is dependent on quality and can be expressed as a polynomial function of q (not presented here).

The foam density (ρ_{foam}) can be calculated from the widely accepted "rule of mixtures" as

$$\rho_{foam} = \rho_l (1 - q) + \rho_g q \tag{Eq 39}$$

where ρ_g is the density of the gas phase. The density of the gas phase is found by means of the real gas law [Eq 32] discussed in the previous section.

A method to determine the laminar, transitional, and turbulent foam flow losses in pipes is presented in Blauer *et al.* (1974). Their method is extended here to include flow in an annulus along with a more refined turbulence criterion. The turbulence criterion for foams is expressed in terms of the Hedstrom number for Bingham-Plastic fluids and is discussed with Bingham-Plastic fluids. In addition, the friction factors for foam flow in straight and coiled tubing are calculated as discussed in the same section.

Pressure Losses in Multiphase Fluids

In general, multiphase fluids refer to a mixture of solid, liquid, and gas. The solid phase is in the form of drilled cuttings, sand, proppants, etc. The liquid phase is usually comprised of water and oil. The gas phase is most often nitrogen, air, or natural gas. Some examples of such multiphase flows are

- annular flows in any drilling operation;
- vertical two-phase flow of oil, gas, and water through production tubing;
- under-balanced drilling operations with nitrified water (a multiphase mixture of water and nitrogen); and
- acid jobs with nitrified acid (a multiphase mixture of nitrogen and dilute acid).

In this document, however, the discussion is limited to a multiphase mixture of liquid and gas without any solid phase. The pressure losses of multiphase fluids in any CT operation are computed using correlations developed for two-phase flow through production tubing. Of the many published correlations on the subject of two-phase flow through production tubing, four multiphase flow models have gained widespread acceptance in the petroleum industry:

- Duns and Ros (1963)
- Hagedorn and Brown (1965)
- Orkiszewski (1967)
- Beggs and Brill (1973)

Although most of these multiphase correlations have been primarily developed for upward, two-phase flow in production tubing alone, they have also been extended to include downward flow through tubing and upward flow through an annulus. As such, these models have been provided in Cerberus for predicting the system pressure losses in CT applications, and each of these models are considered separately a little later in this section.

At this point, it should be emphasized that the segment analysis [Eq 1 through Eq 8] holds true for multiphase fluids as well. However, the fluid properties (density and viscosity) and friction factor computations in Eq 2 through Eq 4 differ considerably from the single-phase calculations. The density and viscosity of multiphase fluids can be evaluated by the simple "rule of mixtures" as

$$\rho_s = \rho_l H_s + \rho_g (1 - H_s) \tag{EQ 40}$$

$$\rho_{ns} = \rho_l H_{ns} + \rho_g (1 - H_{ns}) \tag{EQ 41}$$

$$\mu_s = \mu_l H_s + \mu_g (1 - H_s) \tag{EQ 42}$$

$$\mu_{ns} = \mu_l H_{ns} + \mu_g (1 - H_{ns}) \tag{EQ 43}$$

where H is called the liquid hold-up and represents the volume fraction of the pipe occupied by the liquid phase. The subscripts s and ns refer to slip and non-slip respectively. Similarly, the friction factor is also dependent on the liquid hold-up through the two-phase Reynolds number. It should be noted that liquid properties such as density, viscosity, and surface tension (σ) are evaluated based on the water (ξ_w) and oil (ξ_o) volume fractions in the liquid. Typically, the liquid fluid properties can be expressed as

$$\rho_l = \rho_o \xi_o + \rho_w \xi_w \tag{EQ 44}$$

$$\mu_l = \mu_o \xi_o + \mu_w \xi_w \tag{EQ 45}$$

$$\sigma_l = \sigma_o \xi_o + \sigma_w \xi_w \tag{EQ 46}$$

Here, the subscript o and w refer to oil and water respectively. Many oil system correlations [Baker and Swerdloff (1956); Beggs and Robinson (1975); Chew and Conally (1959); Lasater (1958); Standing (1947); Vasquez and Beggs (1980)] are used to evaluate the properties of oil at downhole conditions, and are not discussed in this document. Clearly, from Eq 40 through Eq 43, evaluation of liquid hold-up is a critical part of the multiphase computations and erroneous system pressure predictions can result if H_s is not estimated accurately. Liquid hold-up can either be measured or calculated, and is dependent on the flow regime. Flow regimes in two-phase flow are classified based on whether the flow is vertical or horizontal. In vertical flow, the flow regimes are usually classified as bubble, slug, froth, transition, and mist flow, whereas in horizontal flow, the usual classification is: segregated, intermittent, transition, and distributed. The description of the flow in these various flow regimes is not given here and can be obtained elsewhere [Beggs and Brill (1975)]. However, these flow regimes can be distinguished from each other by means of dimensionless groups described in the next few subsections where each of the multiphase correlations or models are considered separately.

Duns and Ros Correlation

The Duns and Ros correlation is developed for vertical flow of gas and liquid mixtures in wells. This correlation is valid for a wide range of oil and gas mixtures with varying water-cuts and flow regimes. Although the correlation is intended for use with “dry” oil/gas mixtures, it can also be applicable to wet mixtures with a suitable correction. For water contents less than 10%, the Duns-Ros correlation (with a correction factor) has been reported to work well in the bubble, slug (plug), and froth regions.

Several dimensionless groups are used to distinguish the flow regimes. These dimensionless quantities are given below in consistent units as

$$N_{lv} = v_{sl}\sqrt[4]{(\rho_l / g\sigma)} \text{ (liquid velocity number)} \quad \text{EQ 47}$$

$$N_{gv} = v_{sg}\sqrt[4]{(\rho_l / g\sigma)} \text{ (gas velocity number)} \quad \text{EQ 48}$$

$$N_D = D_e\sqrt{(\rho_l g / \sigma)} \text{ (pipe diameter number)} \quad \text{EQ 49}$$

$$N_l = \mu_l\sqrt[4]{(g / \rho_l\sigma^3)} \text{ (liquid viscosity number)} \quad \text{EQ 50}$$

where σ is the surface tension at the liquid-gas interface, and, v_{sl} and v_{sg} are the superficial liquid and gas velocities respectively. Details on the equations written as a function of the above dimensionless numbers that distinguish various flow regimes can be obtained from the original work of Duns and Ros (1963). In all other multiphase models presented as well, equations determining various flow regimes are not provided and can be obtained from the respective original papers [Hagedorn and Brown (1965); Orkiszewski (1967); and Beggs and Brill (1973)]. Instead only details pertaining to density, viscosity, liquid hold-up, and friction factor in various flow regimes are presented here.

The liquid hold-up calculation in the Duns and Ros (1963) model involves defining a slip velocity (v_s) as

$$v_s = \frac{S}{(\rho_l / \sigma_l g)^{0.25}} \quad \text{EQ 51}$$

where S is the dimensionless slip velocity and is dependent on the flow regime. Expressions for S in bubble and slug flow regimes can be written in terms of Eq 47 through Eq 50 and are not provided here. Once the slip velocity is calculated, liquid hold-up can be found from

$$H_s = \frac{(v_s - v_m) + \left[(v_m - v_s)^2 + 4 v_s v_{sl}\right]^{1/2}}{2 v_s}$$

EQ 52

Here, $v_m = v_{sl} + v_{sg}$ is the multiphase mixture velocity. However, in the mist flow regime, both liquid and gas phases are assumed to move at the same velocity without any slippage and hence, S is zero. In this case, the liquid hold-up is referred to as the non-slip hold-up and is simply

$$H_{ns} = \frac{v_{sl}}{v_m}$$

EQ 53

The multiphase mixture density can now be calculated from Eq 40, Eq 41, Eq 52, and Eq 53 for a particular flow regime.

The mixture viscosity given by Eq 42 and Eq 43 are not used in the Duns and Ros (1963) correlation. Instead, the gas and liquid viscosities are utilized separately depending upon the flow regime. In bubble and slug flow, the Reynolds number is calculated using Eq 11 based on the liquid viscosity (μ_l), and in mist flow, gas viscosity (μ_g) is utilized. Then, the single-phase friction factor is found from the Moody's chart and used to compute the two-phase friction factor. The two-phase friction factor (f_m) can be evaluated from a correlation developed as a function of the single-phase friction factor (Moody's chart), pipe diameter number [Eq 49], and the superficial gas and liquid velocities. In addition, the two-phase friction factor for mist flow is affected by a thin liquid film on the pipe wall and this effect is taken into consideration through a dimensionless quantity called the Weber number. The friction pressure loss is then calculated using Eq 3. The pressure loss due to acceleration [Eq 4] is considered negligible in the bubble and slug flow regimes.

Hagedorn and Brown Correlation

This correlation was developed using data obtained from a 1500 ft vertical well. Tubing diameters ranging from 1 to 2 in. were considered in the experimental analysis along with 5 different fluid types, namely water and four types of oil with viscosities ranging between 10 and 110 cp (at 80°F). The correlation developed is independent of flow pattern with H_s being correlated with the four dimensionless groups given by Eq 47 through Eq 50. As before, once H_s is known, the multiphase mixture density (ρ_s) is calculated using Eq 40. However, for computing the friction pressure loss using Eq 3, the mixture density term in Eq 3 needs to be replaced with

$$\rho_m = \frac{\rho_{ns}^2}{\rho_s}$$

EQ 54

In order to compute the two-phase friction factor, a two-phase Reynolds number is first found using Eq 11 based on the non-slip mixture density (ρ_{ns}), mixture velocity (v_m), and a multiphase mixture viscosity defined as

$$\mu_m = \mu_l^{H_s} * \mu_g^{1-H_s}$$

EQ 55

Based on this two-phase Reynolds number, the friction factor is found from the Moody's friction factor chart. The acceleration term [given by Eq 4] is computed using the slip mixture density and velocity at points 1 and 2 respectively.

Orkiszewski Correlation

The correlation is valid for different flow regimes such as the bubble, slug, transition, and annular mist and is a composite of several published works [Orkiszewski (1967)]. This correlation is limited to two-phase pressure drops in a vertical pipe and is an extension of Griffith and Wallis (1961) work in the bubble and slug flow.

In the bubble flow regime, the liquid hold-up is calculated from

$$H_s = 1 - 0.5\left[1 + \frac{v_m}{v_s} - \sqrt{\left[(1 + v_m / v_s)^2 - 4v_{sg} / v_s\right]}\right]$$

EQ 56

Here, the slip velocity, v_s, is assumed to be a constant value of 0.8 ft/s. As before, the friction factor is calculated from the Moody diagram with the Reynolds number calculated from

$$\mathrm{Re} = \frac{\rho_l v_{sl} D_e}{\mu_l H_s}$$

EQ 57

The acceleration term is considered negligible in the bubble flow regime.

In the slug flow regime, the computation of two-phase fluid properties is somewhat different from the Duns and Ros (1963) and Hagedorn and Brown (1967) correlations. Here, the mixture density is defined in terms of a bubble velocity (v_b) as

$$\rho_s = \frac{\rho_l (v_{sl} + v_b) + \rho_g v_{sg}}{v_m + v_b} + \rho_l \Gamma \qquad \textbf{Eq 58}$$

where Γ is called the liquid distribution coefficient, and is evaluated using the data from the Hagedorn and Brown (1967) model. The value of Γ is calculated from several expressions provided in Orkiszewski (1967) depending upon the liquid phase, mixture velocity, and certain constraints to eliminate pressure discontinuities between flow regimes (not shown here). The bubble velocity in Eq 58 is expressed as a function of the bubble Reynolds number (Re_b) as well as the Newtonian Reynolds number (Re_l) given by

$$\mathrm{Re}_b = \frac{\rho_l v_b D_e}{\mu_l} \qquad \textbf{Eq 59}$$

$$\mathrm{Re}_l = \frac{\rho_l v_m D_e}{\mu_l} \qquad \textbf{Eq 60}$$

The equations for the bubble velocity are not presented here and can be obtained from Orkiszewski's (1967) work. However, it should be mentioned that the calculation procedure for v_b is iterative, and this computed value of v_b is utilized in the friction pressure loss computation. The friction pressure loss in the slug flow regime is somewhat different from the usual form given by Eq 3, and is expressed as

$$\Delta P_f^s = f \; \frac{\rho_l v_m^2 L}{2 g_c D_e} \left[\left(\frac{v_{sl} + v_b}{v_m + v_b} \right) + \Gamma \right] \qquad \textbf{Eq 61}$$

where f is obtained from the Moody's chart based on Eq 60. As in the bubble flow regime, the acceleration term is considered to be negligible and therefore Eq 4 is zero.

In the transition and mist flow regimes, the method presented by Duns and Ros (1963) is used (previous subsection) and is not repeated here.

Beggs and Brill Correlation

Unlike the other three models, the Beggs and Brill (1973) correlation is developed for tubing strings in inclined wells and pipelines for hilly terrain. This correlation resulted from experiments using air and water as test fluids. Liquid hold-up and pressure gradient were measured at various inclinations angles and the correlation was developed from 584 measured tests. As before, details on the flow regimes are omitted here and can be obtained from the original paper. However, it should be mentioned that the flow patterns are distinguished through correlations expressed as a function of the non-slip liquid hold-up given by Eq 53 and a Froude number (dimensionless) written as

$$Fr = \frac{v_m^2}{gD_e}$$

Eq 62

For all flow patterns, the hold-up at any inclination angle (θ) can be expressed in terms of the hold-up when the pipe is horizontal ($\theta = \pi/2$) as

$$H_s(\theta) = H_s(\theta = \pi/2) * \Psi$$

Eq 63

where Ψ is a correction factor accounting for the effect of pipe inclination. The hold-up which would exist at the same conditions in a horizontal pipe is calculated from

$$H_s(\theta = \pi/2) = \frac{c_1 H_{ns}^{c_2}}{Fr^{c_3}}$$

Eq 64

Here, H_{ns} is the non-slip hold-up and, c_i are constants that are determined based on the flow pattern. In addition, the correction factor Ψ is given by

$$\Psi = 1 + C\left[\sin(1.8\phi) - 0.333\sin^3(1.8\phi)\right]$$

Eq 65

where $\phi = \theta - \pi/2$ is the angle from the horizontal.

For vertical upward flow ($\theta = \pi$; $\phi = \pi/2$), Ψ takes the form

$$\Psi = 1 + 0.3C$$

Eq 66

In Eq 65 and Eq 66, C is given by

$$C = (1 - H_{ns})\ln(c_4 H_{ns}^{c_5} N_{lv}^{c_6} Fr^{c_7})$$

Eq 67

with the restriction that $C \geq 0$.

The Reynolds number is calculated from

$$\mathrm{Re}_l = \frac{\rho_{ns} v_m D_e}{\mu_{ns}}$$

EQ 68

where ρ_{ns} and μ_{ns} are the non-slip mixture density and viscosity respectively. The non-slip friction factor (f_{ns}) is found from the Moody's chart and is utilized to find the two-phase friction factor (f_m) as

$$f_m = e^X f_{ns}$$

EQ 69

In Eq 67, X is a function of both H_{nl} and $H_l(\theta)$. Finally, the friction pressure loss can be calculated from Eq 3 by utilizing f_m, ρ_{ns}, and v_m.

General Comments

All four multiphase models discussed have been primarily developed to predict the pressure profile in an upward flow through production tubing. Nonetheless, they have all been generalized to include downward flow through tubing and upward, annular flow. More specifically, when these generalized models are applied to a CT hydraulics simulation (which usually includes both downward flow through the CT and upward flow through the annulus formed between the CT and production tubing/casing), their performance may be affected by errors introduced from the generalization. Therefore, these multiphase correlations should be used with caution keeping their limitations in mind. Furthermore, the friction pressure drop relations presented above apply only to straight tubing and cannot be applied to the length of the coiled tubing on the reel. In fact, the subject of friction pressure drop of multiphase fluids in coiled tubing has been rarely addressed in the literature and correlations that determine the effect of curvature on the friction pressure drop of multiphase fluids need to be developed. Due to the lack of multiphase correlations addressing the effect of reel curvature on the friction pressure drop, Cerberus like all other commercial programs assumes that, on the surface, flow occurs in horizontal, straight tubing just as in the case of gases.

Nomenclature

a,b		constants defined in Eq 24
A_F		cross-sectional area of fill (ft^2)
$C_{i=1,7}$		constants used in Eq 64 and Eq 67
C		parameter used in the Beggs and Brill model
d		inner diameter (ft)
D		outer diameter (ft)
D_e		equivalent diameter (ft)
Dn		Dean number
D_{reel}		reel diameter (ft)
f		Moody's friction factor
Fr		Froude number
g	=	32.174 acceleration due to gravity (ft/s^2)
g_c	=	32.174 Newton's law gravitational constant [$lbm\text{-}ft/(lbf - s^2)$]
h		vertical height of CT segment (ft)
H		liquid hold-up
K		consistency index ($lbf\text{-}s^n/ft^2$)
K_a		annulus constant
L		length of CT segment (ft)
$\dot{m}$		mass flow rate (lbm/s)
n		flow behavior index
N		number of segments
N_{gv}		gas velocity number
N_{lv}		liquid velocity number
N_D		pipe diameter number
N_l		liquid viscosity number
P		pressure (lbf/ft^2)
q		quality of foam
Q		flow rate (ft^3/s)
R		gas constant [(lbf-ft)/(lbm °R)]
Re		Reynolds number
Re_G		generalized Reynolds number for power law fluids

Re_p	particle Reynolds number
S	dimensionless slip velocity
T	temperature (°R)
v	velocity of fluid (ft/s)
V	total volume (ft^3)
X	a function of liquid hold-up used in Eq 69
Z	compressibility factor

Greek Symbols

ΔP	pressure loss (lbf/ft^2)
ε	absolute pipe roughness (ft)
γ	shear rate (1/s)
λ	maximum loading (lbm/ft^3)
μ	viscosity of fluid (lbf-s/ft^2)
ϕ	angle from horizontal (radians)
Ψ	angle correction factor for Beggs and Brill model
ρ	density of fluid (lbm/ft^3)
σ	surface tension (lbf/ft)
τ	shear stress (lbf/ft^2)
τ_w	wall shear stress (lbf/ft^2)
Γ	liquid distribution coefficient for Orkiszewski model
θ	angle of inclination to the vertical (radians)
ξ	volume fraction

Subscripts

1, 2	points 1 and 2 of segment shown in Figure 1
a	acceleration
cr	critical
C	casing / production tubing
CT	coiled tubing
e	effective
f	friction
g	gas
h	hydrostatic
l	liquid
m	mixture
ns	non-slip
o	oil
p	plastic
s	slip
sg	superficial gas
sl	superficial liquid
ST	straight tubing
T	CT / tubing
w	water
y	yield

Superscripts

s	segment

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