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The Effective Force

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Contents

Summary

In the “Basic Tubing Forces Model” Tech Note, the effective force, F_e, is introduced as a convenience in calculating axial force within a segment of CT. Furthermore, the effective force is claimed to be the decisive factor affecting helical buckling of tubing, as opposed to the real axial force.

The purpose of the following discussion is to provide a more intuitive understanding of the effective force and its relationship to real axial force and to substantiate the claims made as to its relationship with helical buckling.

The Effect of External Pressure

Consider a tubular (CT, riser, tubing, casing) in the configuration below bent downhole, the tubular terminates at AB. Fluid has access to the termination AB and imparts a pressure force, *f*, which subjects the rod to compression.

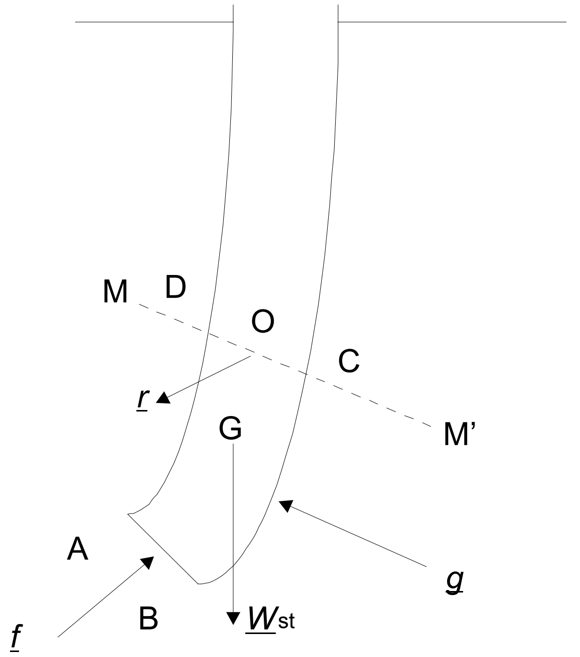

FIGURE 1 Effect of Outside Pressure on Tubular

Now consider any cross-section MM'. The forces acting on the portion of the rod below MM' are as follows:

1. A pressure force *f*, that subjects the tubular to a compression.
2. The weight *Wst* of the portion of the rod below MM' (the weight is applied at the center of gravity of the section).
3. The reaction *r* of the portion of the rod above MM' on the portion below MM'.
4. The resultant *g* of the pressure forces acting on the lateral surface of the rod. The resultant *g* is not zero because arc BC is longer than arc AD. Consequently the area at the vicinity of BC is greater than the area at the vicinity of AD; therefore pressure forces in the direction of *g* are greater than the pressure forces in the direction opposite to *g*. Thus the resultant pressure acting on the lateral surface is as shown in Figure 1 by the vector *g*.

Let O be the center of the cross section MM'. The bending moment, M, at the cross-section MM' is:

$$\underline{M} = \underline{M}o(\ \underline{f}\ ) + \underline{M}o(\ \underline{g}\ ) + \underline{M}o(\ \underline{W}st\ ) \text{ , noting that } \underline{M}o(\ \underline{r}\ ) = 0 \qquad \textbf{Eq 1}$$

It might seem that the determination of the force $\underline{g}$ is difficult, as its magnitude depends on the shape of the bent tubular. $\underline{g}$ may be determined in the following manner. Consider the portion ABCD immersed in the fluid, from Archimedes' law, the resultant forces of $\underline{f}$, $\underline{g}$ and $\underline{r}$ is equal to the weight of the fluid displaced by the portion of the rod below MM', see Figure 2, where $\underline{W}o$ is an upward force applied at the center of gravity G:

$$\underline{f} + \underline{g} + \underline{r} = -\ \underline{W}o \qquad \textbf{Eq 2}$$

Note that $\underline{W}o$ is in the opposite direction of $\underline{W}st$, hence the minus sign.

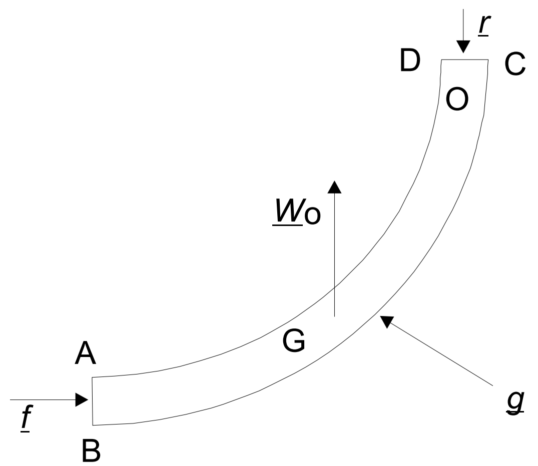

FIGURE 2 Free Body Diagram showing the Effect of External Fluid

Taking moments of forces at O, we have:

$$\underline{M}o(\ \underline{f}\ ) + \underline{M}o(\ \underline{g}\ ) = -\ \underline{M}o(\ \underline{W}o) \qquad \textbf{Eq 3}$$

Eq 1 and Eq 3 leads to:

$$\underline{M} = -\ \underline{M}o(\ \underline{W}o\ ) + \underline{M}o(\ \underline{W}st) \qquad \textbf{Eq 4}$$

which may be written as:

$$\underline{M} = \underline{M}o(\ \underline{W}st - \underline{W}o\ ) \qquad \textbf{Eq 5}$$

If the cross-section MM' were at AB, then $\underline{W}st = \underline{W}o$ (the weight of the rod $\underline{W}st$ and the weight of the displaced fluid $\underline{W}o$, both below AB would be zero). Thus the bending moment M is zero in spite of the compression imparted by the pressure force.

If the cross section MM' is not at AB, then the bending effect of the pressure force $\underline{f}$ is essentially canceled by the bending effect of the pressure force $\underline{g}$.

From the above arguments and by comparing Eq 5 and Eq 1, the following important conclusions may be drawn:

From a bending standpoint, the system in Figure 1 behaves as:

1. As if the compressive force at AB were zero not f.
2. As if the weight of any portion of the submerged rod were replaced by the weight minus the weight of the displaced fluid.

The effect of external pressure, on bending, is to introduce a - f force, thus the effective compressive force at AB is zero. Therefore compression due to the presence of fluid around a pipe freely suspended in a well cannot cause buckling.

The effective and actual longitudinal forces at a distance x above AB are as follows, considering tension as positive and compression as negative:

Effective - $(\xi_{st} - \xi_o)\,x$

Actual - $-f + \xi_{st}\,x$

where ξ_{st} is the weight of the rod per unit length, and ξ_o is the weight of the displaced fluid per unit length. The distribution of these forces along the length of the tubular are shown in Figure 3, in which it has been assumed that the top of the tubular is at the fluid surface. The length of the tubular is L, as $f = \xi_o L$, both the effective and the actual force are equal at the surface, and equal to the weight of the rod in the fluid, i.e. the suspended weight.

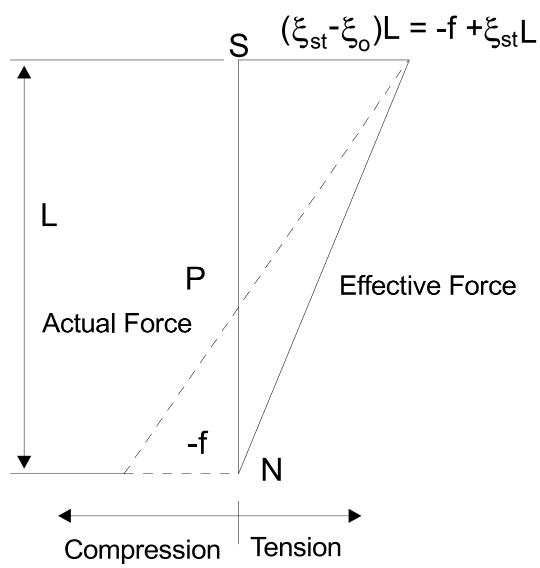

FIGURE 3 Distribution of Effective and Actual Forces along a Tubular, immersed in Fluid.

If now there is an applied compressive force $\underline{F}$ on AB, the actual compressive force at AB would be $\underline{F} + \underline{f}$. Thus the distribution of the effective and actual forces can be described by Figure 4. Note, N is denoted by the neutral point, where the effective force is zero and is very important in statics of bending and dynamics of transverse vibrations.

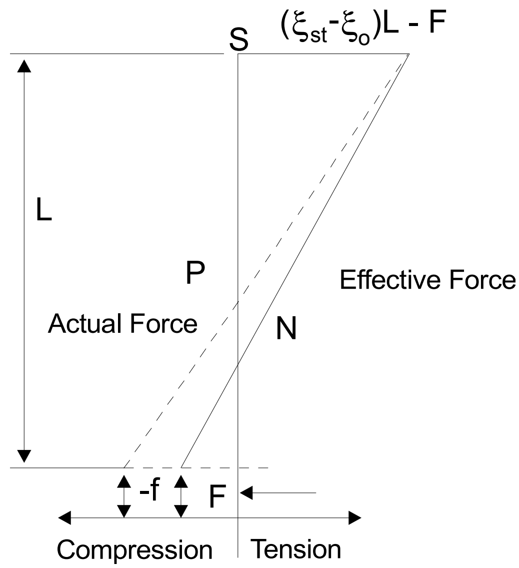

FIGURE 4 Distribution of Effective and actual Forces along a Tubular, acted upon by a compressive force F, immersed in fluid.

Effect of Internal pressure

Now consider the effect of fluid pressure inside the tubular, the reasonings and conclusions will be similar to those previously reached. Figure 5 and Figure 6, deal with the effects of fluid inside the tubular and are analogous to Figure 1 and Figure 2.

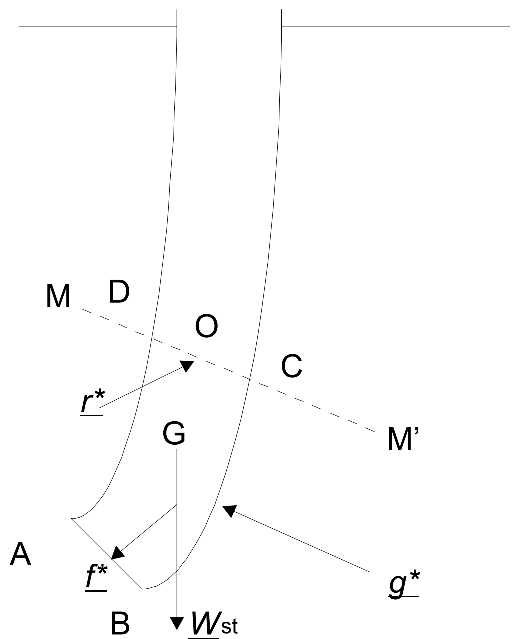

FIGURE 5 Effect of Internal Pressure

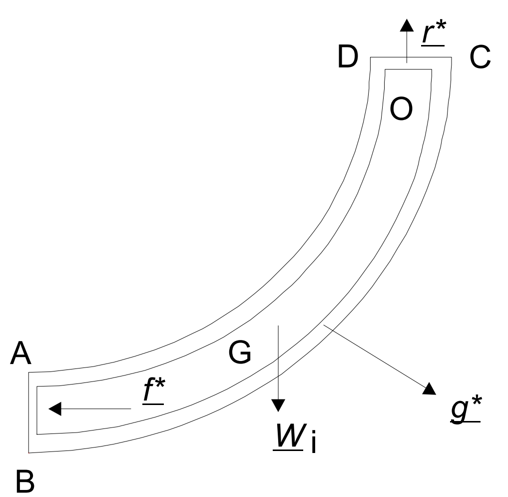

FIGURE 6 Effect of Internal Pressure

The tubular terminates at AB, the pressure force $\underline{f}$* subjects the pipe to tension, but its bending effect is essentially compensated by the pressure force $\underline{g}$*, on the lateral surface of the inside of the tubular below MM'. The portion of the pipe below MM' is subjected to forces $\underline{f}$*, $\underline{W}st$, $\underline{g}$* and $\underline{r}$*. The bending moment M at the section MM' is:

$$\underline{M} = \underline{M}o(\underline{f}^*) + \underline{M}o(\underline{g}^*) + \underline{M}o(\underline{W}st) \quad \text{Eq 6}$$

Now consider the fluid inside the pipe, we get

$$\underline{f}^* + \underline{g}^* + \underline{r}^* = \underline{W}i \quad \text{Eq 7}$$

where, *Wi* is the weight of the fluid inside the tubular below MM'. The force *Wi* is applied at the center of gravity and directed downward. From this, taking moments at O:

$$\underline{M}o(\underline{f}^*) + \underline{M}o(\underline{g}^*) = \underline{M}o(\underline{W}i) \quad \text{Eq 8}$$

Substituting Eq 8 in Eq 6 gives:

$$\underline{M} = \underline{M}(\underline{W}st + \underline{W}i) \quad \text{Eq 9}$$

From Eq 9 and Eq 6, the following conclusions may be drawn:

From the bending (or straightening) standpoint, the system of Figure 6 behaves:

1. As if the tension of the lower termination were zero and not $\underline{f}$*.
2. As if the weight of any portion of the pipe was replaced by that weight plus the weight of the corresponding inside fluid.

The effective and actual longitudinal forces at a distance x from AB are as follows:

Effective - $(\xi_{st} + \xi_i)\,x$

Actual - $+f + \xi_{st}\,x$

where ξ_{st} is the weight of the rod per unit length, and ξ_i is the weight of the fluid inside the tubular per unit length. The distribution of these forces along the length of the pipe is shown in Figure 7, which is analogous to Figure 3.

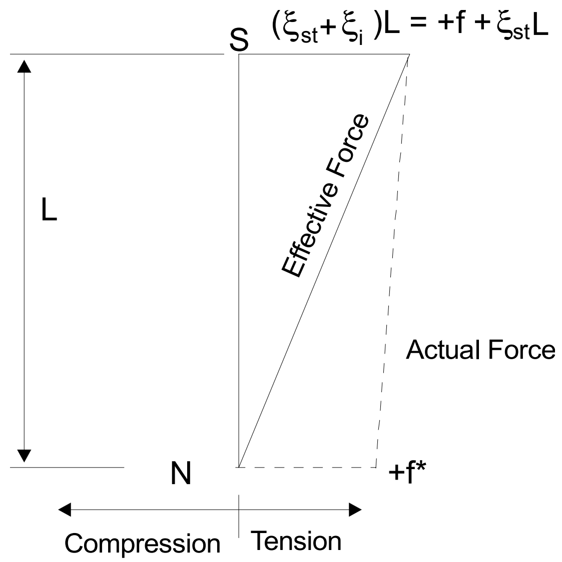

FIGURE 7 Distribution of Effective and Actual Forces along a pipe with Internal Fluid

If now a compressive force *-F* is applied at AB, the actual force at AB is *-F + f** and the effective force is *-F*. Thus at AB, the effective force is compressive which could make the pipe buckle, inspite of the fact that the actual force is tensile (if *-F + f** > 0) . This shows that a tensile force due to the presence of fluid inside the pipe cannot prevent buckling. Figure 8 illustrates the distribution of the effective and actual force in this case.

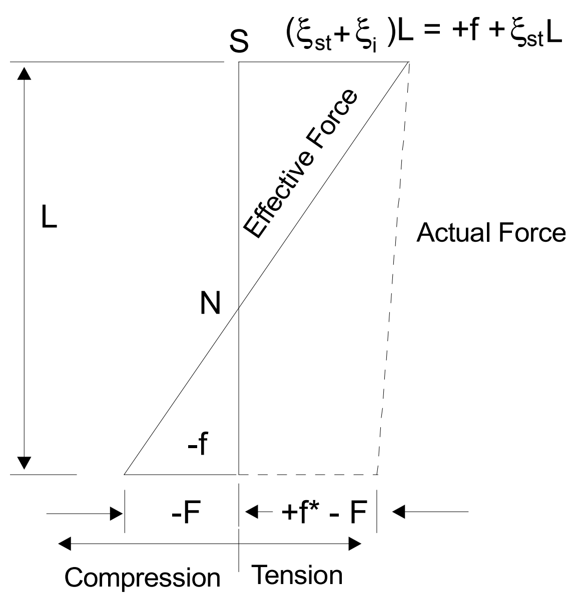

FIGURE 8 Distribution of Effective and Actual Forces along a pipe with Fluid inside and with an Applied Compressive Force.

Thus, it is seen that the effect of fluid internal and external to a bent tubular is to change the effective weight per unit length to

$$\xi_{eff} = \xi_{st} + \xi_i - \xi_o \qquad \textbf{EQ 10}$$

and if there is an applied compressive force *-F*, a resultant force *-f* due to the fluid external to the tubular and a result force *f** due to the fluid internal to the tubular at any section in question. Then the actual force is *-F - f + f**, the effective force is *-F*.

Hence the effect of fluid is to subtract a “fictitious force” *-f + f** from the actual force. Furthermore, compression due to the presence of fluid around a pipe freely suspended in a well cannot cause any buckling. Conversely, a tension due to the presence of fluid inside the pipe cannot prevent buckling.

An Alternative Derivation of the Effective Force

The usual equation for beam buckling in a plane has the form:

$$EI\, y'''' + F\, y'' = 0 \qquad \textbf{EQ 11}$$

where y(z) is the buckled lateral displacement and F is the constant axial compressive force acting on the beam column and the ‘ denotes differentiation wrt z, the axial co-ordinate along the beam. As, in general EI y’’’’ is the lateral loading per unit length on the beam, it is clear that the elementary buckling problems amount to solving beam problems with a lateral load per unit length amounting to -F y’’ . Consequently, to find the influence of external or internal pressure on buckling, it is necessary to find the lateral load imposed by pressure on the beam for a generic lateral displacement of the beam.

Consider a right hollow cylinder (not necessary with a circular cross section) subjected to internal and external pressures, as shown in Figure 9. Take a short length, dz, between two cross sections. Now bend the cylinder about an arbitrary axis. Let y be the perpendicular distance from the bending axis as shown in Figure 9 where the inside boundary of the cylinder is shown. It is straight-forward to show that there is no net force developed on the length dz by applying a uniform internal pressure, *pi* , to the inside surface prior to bending.

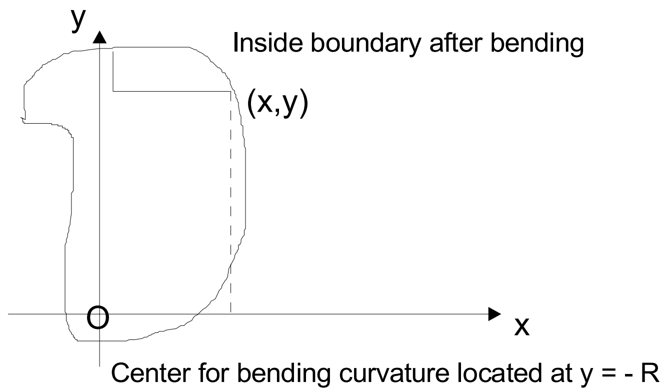

FIGURE 9 Bending of an Arbitrary Cross-Section

When the axis of the cylinder is bent to a given radius of curvature, *R*, the axial strain of the cylinder is given by y/R so that the length, which was dz before bending is $(1 + y/R)dz$ after bending. The forces *Fx* and *Fy* and moment *Mo*, about O, caused by the internal pressure acting on the deformed inside boundary of the cylinder are:

$$F_x = p_i\, dz \oint (1 + y/R)dy = 0$$

$$F_y = p_i\, dz \oint (1 + y/R)dx = p_i\, dz\, A_i/R$$

$$M_o = p_i\, \mathrm{dz} \oint (1 + y/R)\,(xdx - ydy) = (p_i\, dz/R) \oint yx\, dx = p_i\, dz\, A_i \overline{X_i}/R$$

where A_i is the area described by the tube inside boundary cross-section and $\overline{X_i} = \left(\oint xdA_i\right)/A_i$ is the x co-ordinate of the centroid of the inside cross-sectional area with respect to the origin O.

These results show that the lateral force associated with p_i is perpendicular to the bending axis, directed away from the center of bending curvature, equal to $dz\,(pi\, A_i/R)$ and has it's line of action through the centroid of the inside cross sectional area.

A similar derivation using the outside cross sectional area, A_o, and the external pressure p_o, shows that the lateral force associated with p_o is perpendicular to the bending axis, directed toward the center of bending curvature, equal to $dz\,(p_o\, A_o/R)$ and has it's line of action through the centroid of the outside cross section area (located at *Xo*). Consequently, the net force per unit length, F_p, owing to p_o and p_i is:

$$F_p = (p_i\, A_i - p_o\, A_o) / R$$

The moment per unit length, Mp, about O is

$$M_p = (p_i\, A_i\, X_i - p_o\, A_o\, X_o)/R$$

The above results can be used to extend the equation for buckling. Note that, for small displacements, 1/R equals -y'' so that the governing beam-column equation becomes:

$$EI\ y'''' + (F + p_i A_i - p_o A_o)\ y'' = 0$$

and the equation for the effective force becomes:

$$F_{eff} = F + p_i A_i - p_o A_o$$

In this equation, *F* is the actual compressive force on the cylinder. In the determination of *F*, all influences such as end pressure must be accounted for.

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