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Fluid Drag on a Tool

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Summary

Fluid drag on the tool is the resultant force exerted on the tool as a result of motion of the fluid on the external surface of the tool. This resultant force or total fluid drag is usually an upward, lifting force on the tool, and is comprised of three components, namely:

- a shear force acting on the tool surface along the length of the tool (commonly referred to as viscous drag, F_s);
- a normal force due to the pressures at tool ends acting perpendicular to the tool face in the axial direction (commonly referred to as pressure drag, F_P); and
- an upward force resulting from hydrodynamic effects of the tool shape (F_a) in the flow path.

Mathematically, the total fluid drag on the tool, F_T, can be written as a sum of all three components,

$$F_T = F_s + F_P + F_a \qquad \textbf{EQ 1}$$

In this document, all three components of fluid drag are discussed in some detail.

Introduction

Figure 1 illustrates the flow geometry in which a cylindrical tool (length, L_T; diameter, d_T) is suspended from a wireline in a production tubing or casing. As the fluid flows upwards from Section 1, it is subject to a sudden contraction at the bottom or front end of the tool. Section 'vc' refers to the *vena contracta* at which the fluid velocity is maximum due to the contraction and is typically expressed in terms of a contraction coefficient, C_c. (The contraction coefficient C_c is defined as the ratio of the cross-sectional area at the contraction to the cross-sectional area at Section 2). The pressure loss associated with this sudden contraction (ΔP_{LC}) contributes to the total drag or upward force exerted by the fluid on the tool. In addition, as the fluid travels along the length of the tool from Section 'vc' towards the rear or top end of the tool a certain amount of energy is lost due to the effects of friction. This frictional pressure loss is related to the shear stress acting on the tool surface along its length and, as mentioned earlier, contributes to the viscous or shear drag. Finally, as the fluid exits the rear or top end of the tool (between Sections 2 and 3), the fluid is subjected to a sudden expansion (ΔP_{LE}) and the drag associated with the expansion pressure losses must be taken into account to obtain the total fluid drag on the tool.

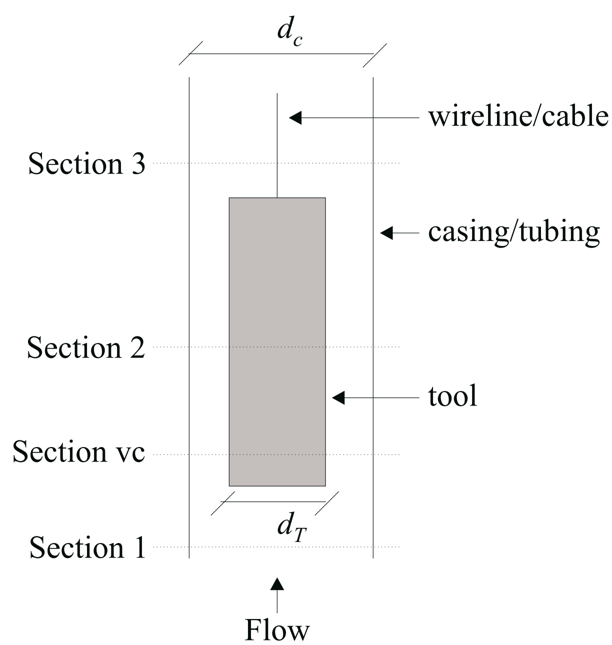

FIGURE 1 Flow geometry depicting annular flow at the tool

Viscous/Shear and Pressure Drag

The shear stress, τ, acting on the tool surface can be approximated as[1]

$$\tau = \left(\frac{d_c - d_T}{4}\right)\frac{\Delta P_f}{L_t}$$

EQ 2

where d_c is the diameter of the casing/tubing and ΔP_f is the friction pressure loss across the tool length, L_T [Eq 2 and all subsequent equations in this document are expressed in consistent units unless otherwise noted]. The friction pressure loss can be obtained from the relation

$$\Delta P_f = f_M \frac{\rho v_i^2 L_T}{2 g_c D_e}$$

EQ 3

Here, ρ is the density of the fluid, v_i is the velocity of the fluid at Section i, D_e is the equivalent diameter of the conduit, and f_M is the Moody's friction factor which is a function of the Reynolds number (Re) of the flow. For Newtonian fluids, Re can be written as

$$\mathrm{Re} = \frac{\rho v_i D_e}{\mu}$$

EQ 4

where μ is the viscosity of the fluid. For internal flows, the flow can be classified as either laminar, transitional, or turbulent depending on the magnitude of the Reynolds number. The flow is laminar if Re is less than or equal to a critical value of 2100. A transitional flow is observed between Reynolds numbers of 2100 and 4000. If Re is greater than 4000, then the flow is turbulent. The friction factor in laminar flows is given by the well-known relation

$$f_M = \frac{64}{\mathrm{Re}}$$

EQ 5

In turbulent flow, f_M for smooth (Blasius equation) and rough pipes (Colebrook equation) can be expressed as[1]

$$f_M = \frac{0.3164}{\mathrm{Re}^{0.25}} \quad \text{(smooth pipes)}$$

EQ 6

$$f_M = \left[\frac{1}{2}\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f_M}} + \frac{\varepsilon / D_e}{3.715}\right)\right]^2 \quad \text{(rough pipes)}$$

EQ 7

In Eq 7, ε is the absolute roughness of the tubing and is approximately equal to 0.00186 in. for commercial steel pipes. The viscous or shear drag (F_s) is obtained from the relation

$$F_s = \tau \cdot \pi d_T L_T \qquad \text{EQ 8}$$

In addition to the roughness, factors such as eccentricity and equivalent diameter will have an effect on the magnitude of the fluid drag. The friction pressure loss in a eccentric pipe ($\Delta P_f\big|_{ecc}$) may be expressed as a function of the friction pressure loss in concentric pipes ($\Delta P_f\big|_{conc}$)[3] as

$$\Delta P_f\big|_{ecc} = \Psi \cdot \Delta P_f\big|_{conc} \qquad \text{EQ 9}$$

where Ψ is the eccentricity factor and is given by

$$\Psi = 1 - 0.072 E_c \left(\frac{d_T}{d_c}\right)^{0.8454} - \frac{3}{2} E_c^2 \left(\frac{d_T}{d_c}\right)^{0.1852} + 0.96 E_c^3 \left(\frac{d_T}{d_c}\right)^{0.2527} \quad \text{(laminar flows)} \qquad \text{EQ 10}$$

$$\Psi = 1 - 0.048 E_c \left(\frac{d_T}{d_c}\right)^{0.8454} - \frac{2}{3} E_c^2 \left(\frac{d_T}{d_c}\right)^{0.1852} + 0.285 E_c^3 \left(\frac{d_T}{d_c}\right)^{0.2527} \quad \text{(turbulent flows)} \qquad \text{EQ 11}$$

In Eq 10 and Eq 11, E_c is the eccentricity defined as $E_c = 2\delta / (d_c - d_T)$ where δ is the distance between the centers of the outer tubing and inner tool. In general, the value of Ψ is around 0.6 for a fully eccentric tool or cable in turbulent flows. Hence, the magnitude of the friction pressure loss for a fully eccentric annulus is only about 60% of the corresponding magnitude in a concentric case.

Several definitions are available for the equivalent diameter, D_e in the literature, and, in general, can be written as

$$D_e = K_a (d_C - d_T) \qquad \text{EQ 12}$$

where K_a is an annulus constant. The most common values of K_a are 1.0 and 0.816. When K_a = 1, Eq 12 reduces to the hydraulic diameter (defined as four times the cross-sectional flow area divided by the wetted perimeter), and for a K_a value equal to 0.816, Eq 12 becomes the slot flow representation of an annulus flow. It should be noted that this slot flow approximation yields accurate results only for d_T / d_c ratios greater than 0.3 (small annular areas) and should be used with caution.

The normal force (F_p) resulting from the difference in pressure at the two ends of the tool can be written as

$$F_P = \Delta P_f \cdot A_T \qquad \text{EQ 13}$$

where A_T is the cross-sectional area of the tool.

Effects of Tool Shape on Fluid Drag

Application of Bernoulli's equation between Sections 1 and 'vc' yields the following expression for the pressure loss without accounting for the irreversible losses due to the contraction (ΔP_c)

$$\Delta P_C = \frac{1}{2}\frac{\rho v_{vc}^2}{g_c}\left[1-\left(\frac{C_c A_2}{A_1}\right)^2\right] \qquad \text{EQ 14}$$

Here, v_{vc} is the velocity at the *vena contracta* ($v_{vc}=Q / C_c A_2$), A_1 and A_2 are the cross-sectional areas at Sections 1 and 2 respectively. The contraction coefficient C_c is determined by a curve fit to the data obtained for flow of water through a sudden contraction[4], and is given by

$$C_c = 0.6349 - 0.1697\left(\frac{A_2}{A_1}\right) + 0.5145\left(\frac{A_2}{A_1}\right)^2 \qquad \text{EQ 15}$$

The irreversible pressure losses due to the contraction (ΔP_{LC}) are given by

$$\Delta P_{LC} = \left(\frac{1}{C_c}-1\right)^2 \frac{\rho v_2^2}{2 g_c} \qquad \text{EQ 16}$$

where, v_2 is the velocity of the fluid at Section 2 and g_c is the Newton's law conversion constant for the British system of units. At the rear end of the tool, however, there is a reversible pressure gain due to the sudden expansion from Section 2 to 3. The expression for this pressure gain is similar to Eq 14 and is given by

$$\Delta P_E = \frac{1}{2}\frac{\rho v_2^2}{g_c}\left[1-\left(\frac{A_2}{A_3}\right)^2\right] \qquad \text{EQ 17}$$

where A_3 is the cross-sectional area at Section 3. The irreversible pressure losses associated with the sudden expansion (ΔP_{LE}) on the rear end of the tool can be written as

$$\Delta P_{LE} = K \frac{\rho v_2^2}{2g_c}$$

EQ 18

In Eq 18, K is referred to as an expansion coefficient and is usually determined experimentally. However, it can also be represented as[2,4]

$$K = \left[1 - \left(\frac{A_2}{A_3}\right)^2\right]^2$$

EQ 19

If the length of the tool is small such that the fluid stream expands from the *vena contracta* into Section 3, then the irreversible pressure losses due to both contraction and expansion can be calculated as

$$\Delta P_L = \left[\frac{A_3 - C_c A_2}{C_c A_2}\right]^2 \frac{\rho v_3^2}{2g_c}$$

EQ 20

The force exerted by the fluid on the tool due to effects of tool shape (F_a) is simply the product of the pressure loss term (ΔP_a) and area of the tool, A_T, i.e.,

$$F_a = \Delta P_a \cdot A_T$$

EQ 21

In Eq 21, $\Delta P_a = \Delta P_C - \Delta P_E + \Delta P_{LC} + \Delta P_{LE}$ for a long tool and $\Delta P_a = \Delta P_C - \Delta P_E + \Delta P_L$ for a short tool where the irreversible losses due to contraction and expansion are given by Eq 20.

It should be noted that the above analysis is only valid for incompressible flow of a Newtonian fluid. However, this analysis can be extended to include both incompressible and compressible fluids. For incompressible, non-Newtonian fluids, the above method to compute the fluid drag on tool can be easily applied by accounting for the friction pressure losses appropriately. On the other hand, for compressible fluids such as gases and multiphase fluids, the above analysis must be used with caution since density of the fluid can no longer be treated as a constant.

Example

Tool Data: length (L_T) = 21.33 ft, diameter (d_T) = 3.125 in.

Tubing Data: inner diameter (d_c) = 4.78 in.

Fluid Data: bottomhole density (ρ) = 5.754 ppg, bottomhole viscosity (μ) = 0.45 cp, formation volume factor (B_o) = 1.41, surface flowrate (Q) = 517 BPD

The velocity at downhole conditions is found by using the relation

$$v_1 = \frac{QB_o}{A_1} = 9.25 \text{ ft/s}, \quad v_2 = \frac{QB_o}{A_2} = 16.14 \text{ ft/s} \qquad \text{EQ 22}$$

Here, $A_1 = \pi d_c^2 / 4$ and $A_2 = \pi(d_c^2 - d_T^2)/4$ respectively. The Reynolds number is found from Eq 4 with the equivalent diameter $D_e = d_c - d_T$ = 1.655 in. (hydraulic diameter). Substituting in Eq 4, Re = 316961. Clearly, the flow is turbulent. The Moody's friction factor is now found using Blasius equation for smooth pipes (Eq 6) which gives f_M = 0.01332. The friction pressure loss across the length of the tool (ΔP_f) can now be calculated from Eq 3 (assuming E_c = 0, concentric case; and K_a = 1, hydraulic diameter) as

$$\Delta P_f = 2.5 \text{ psi} \qquad \text{EQ 23}$$

The viscous or shear drag on the tool surface (F_s) is obtained with the aid of Eq 2 and Eq 8 as

$$F_s = \left(\frac{d_c - d_T}{4}\right)\Delta P_f \cdot \pi d_T = 10.13 \text{ lbf} \qquad \text{EQ 24}$$

The pressure drag on the tool is given by Eq 13 with $A_1 = \pi d_T^2 / 4$. Thus,

$$F_p = 19.14 \text{ lbf} \qquad \text{EQ 25}$$

Now, the pressure losses due to the contraction and expansion are calculated to determine the effect of area change on the fluid drag on tool. For this example, A_2 / A_1 = 0.57267, and using Eq 15, the contraction coefficient C_c = 0.70644. The velocity at the *vena contracta* is given by $V_c = V_2 / C_c$ = 22.846 ft/s. The acceleration pressure loss due to contraction and expansion can now be calculated from Eq 14 and Eq 17 as

$$\Delta P_C = 4.1\,\text{psi}\,, \quad \Delta P_E = 0.81\,\text{psi} \qquad \textbf{Eq 26}$$

The irreversible pressure losses to due sudden contraction and expansion are found from Eq 16 and Eq 18 as

$$\Delta P_{LC} = 0.21\,\text{psi}\,, \quad \Delta P_{LE} = 0.54\,\text{psi} \qquad \textbf{Eq 27}$$

It should be noted that while calculating the irreversible expansion losses, the coefficient K is found by considering $A_3 \approx A_1$. Thus, from Eq 26 and Eq 27, the total pressure loss due to the area change (ΔP_a) including irreversible losses becomes

$$\Delta P_a = 4.1 - 0.81 + 0.21 + 0.54 = 4.04\text{ psi.} \qquad \textbf{Eq 28}$$

The contribution to the drag from the acceleration components can be determined from Eq 21 as

$$F_a = 30.98\text{ lbf} \qquad \textbf{Eq 29}$$

Thus, the total drag on the tool can be computed by summing the three contributions (Eq 24, Eq 25, and Eq 29), i.e.,

$$F_T = 10.13 + 19.14 + 30.98 = 60.25\text{ lbf} \qquad \textbf{Eq 30}$$

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